ME1020
Mechanical vibrations

Lecture 1

Introduction & revision

Objectives

- Describe the course including the class policy, topics, learning outcomes, etc.
- Explain the concepts of vibration
- Revised basic concepts and maths

Instructors’ & class information

- **Instructor:** S.C. Fok, PhD
- **Office:** Room 222 (Zone 4) Currently under quarantine
- **Office hours:** Wednesday 14:00 – 16:00;
  Thursday 14:00 – 16:00
- **Email:** saicheong.fok@scupi.cn
- **TA:**
  - He Tingting, Email: 1415696650@qq.com

Lectures:

Zone 3-309 on Friday 8:15 - 11:00

Learning resources

- **Textbook:**

Engineering Vibration 4th Edition, D.J. Inman, Pearson Higher Ed., ISBN–9780273768449

Additional references and supplementary notes (if needed) will be posted on Blackboard

Course objective

The aim of this course is to:

- introduce the foundations of vibration theory and its applications to the analysis and design of mechanical systems
- Utilize computer-aided tools in vibration

Skill Set
Design including analysis & communication; utilization of computer-aided tools in vibration control

Course overview

No.	Topics
1	Free response
2	Response to harmonic excitation
3	General forced response
4	MDOF systems
5	Vibration suppression
6	Vibration measurements & machine condition monitoring

Course learning outcomes

At the completion of this course, students will be able to:

- Evaluate the free and forced responses of single and multiple degree of freedom systems;
- Develop solutions to suppress the vibrations or utilize vibrations for machine condition monitoring
- Utilize computer tools to analyze machine vibrations

Assessments & Grading

Description	Percentage
Assignments / quizzes / participation	20%
Lab / project	20%
Midterm	30%
Final exam	30%

- Students must follow/satisfy the rules/requirements stated in the assessment items

Class policy

- Attendance at all scheduled class section is expected
- Students who are absent should inform the instructor in a timely manner. They are responsible to acquire class materials and assignment notes from their classmates
- All assignments must be neatly completed and submitted on time. Only in exceptional circumstances where supporting evidence is supplied and discussed with the instructor, in a timely manner, will

  (a) extensions be granted

  (b) late work be accepted without penalty (Penalty will be decided by the instructor based on the circumstances)
- Academic misconduct is not tolerated
- All disputes and appeal of grades must be filed through a written process

Class policy

Blackboard

- Important information concerning this unit of study is placed on Blackboard, accessible via https://learn.scupi.cn/
- It is your responsibility to access on a regular basis the Blackboard site for
  - Course materials,
  - Course announcements,
  - Online quizzes, assignments, projects, etc.
- You should also check your SCUPI email regularly

The big picture

Rigid Bodies

Deformable Bodies

Fluids

Mechanics:

The study of how bodies react to forces acting on them

Dynamics:

Study of bodies in motion

Statics:

Study of stationary bodies

Vibrations:

Study the relationship between forces and oscillatory motion of mechanical systems

What is the course about?

- This course deals with the harmonic response analysis of dynamic systems so that the vibration can be suppressed through appropriate means

Importance of Vibration

Tacoma Narrows Bridge 1940

Importance of Vibration

Vibration characteristics

- Oscillatory motion
- Wasted energy
- A major cause of premature component failure
- Cause of noise which contributes to discomfort
- To prevent the vibration failures, we need to

a) Understand the phenomenon and the basic concepts;

b) Determine the excitation sources and then find ways to solve the problem

Oscillatory motion

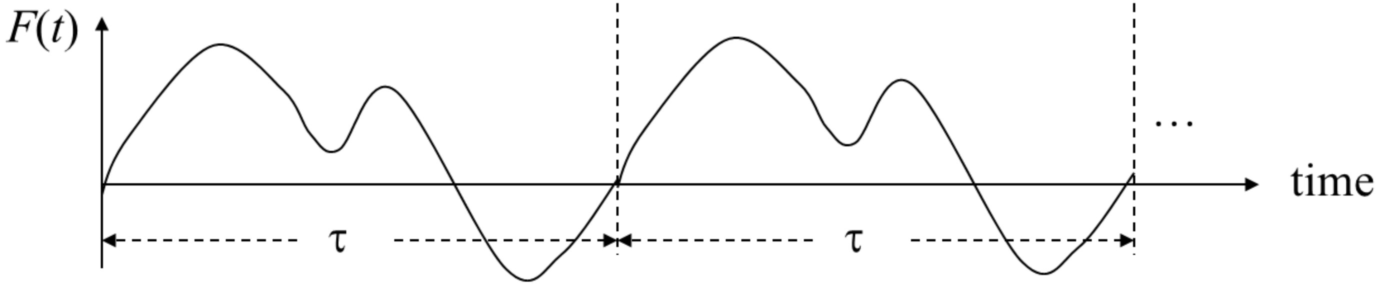
Oscillation motion can be

- ❖ Regular and periodic (e.g. simple harmonic motion)
- ❖ Random and irregular (e.g. earthquake)

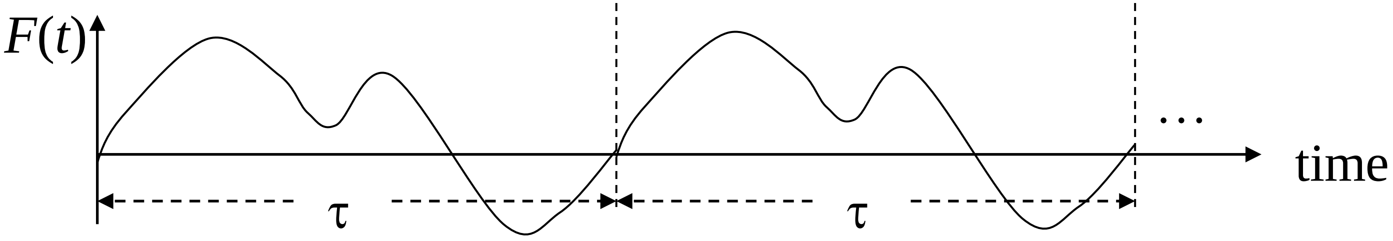
- Periodic motion: motion is repeated after equal intervals of time (called period)

Period

Motion represented by mathematical function $F(t+\tau) = F(t)$



Periodic excitation



A general periodic force repeats itself after a cycle
The time to complete one cycle is called the period $\tau = 1/f = 2\pi/\omega$
The force can be described in the time domain by:

- Peak value A
- Average value $\bar{x} = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)dt$
- Mean square value $\bar{x}^2 = \lim_{T\to\infty} \frac{1}{T}\int_0^T x^2(t)dt$
- Root mean-square value $x_{rms} = \sqrt{\bar{x}^2}$

Simple harmonic motion

- Displacement:

$$x = A\sin\theta = A\sin\omega\, t$$

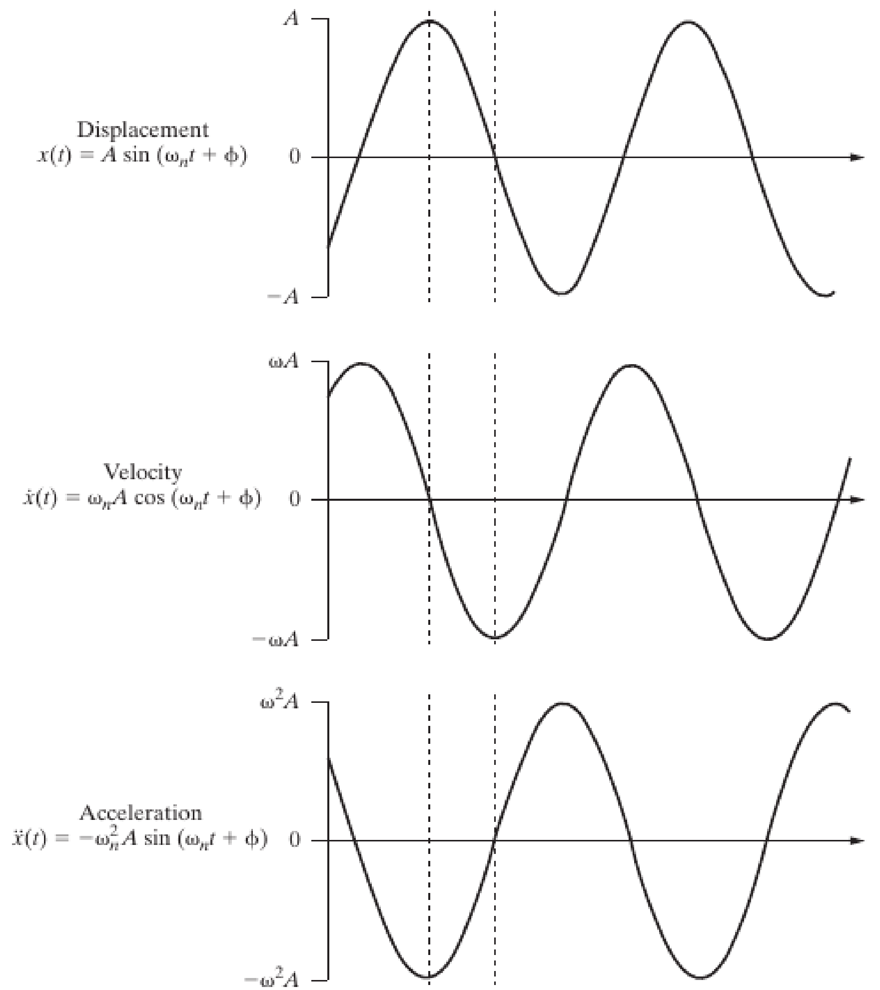
- Velocity:

$$\dot{x} = A\cos\theta = \omega A\cos\omega\, t$$

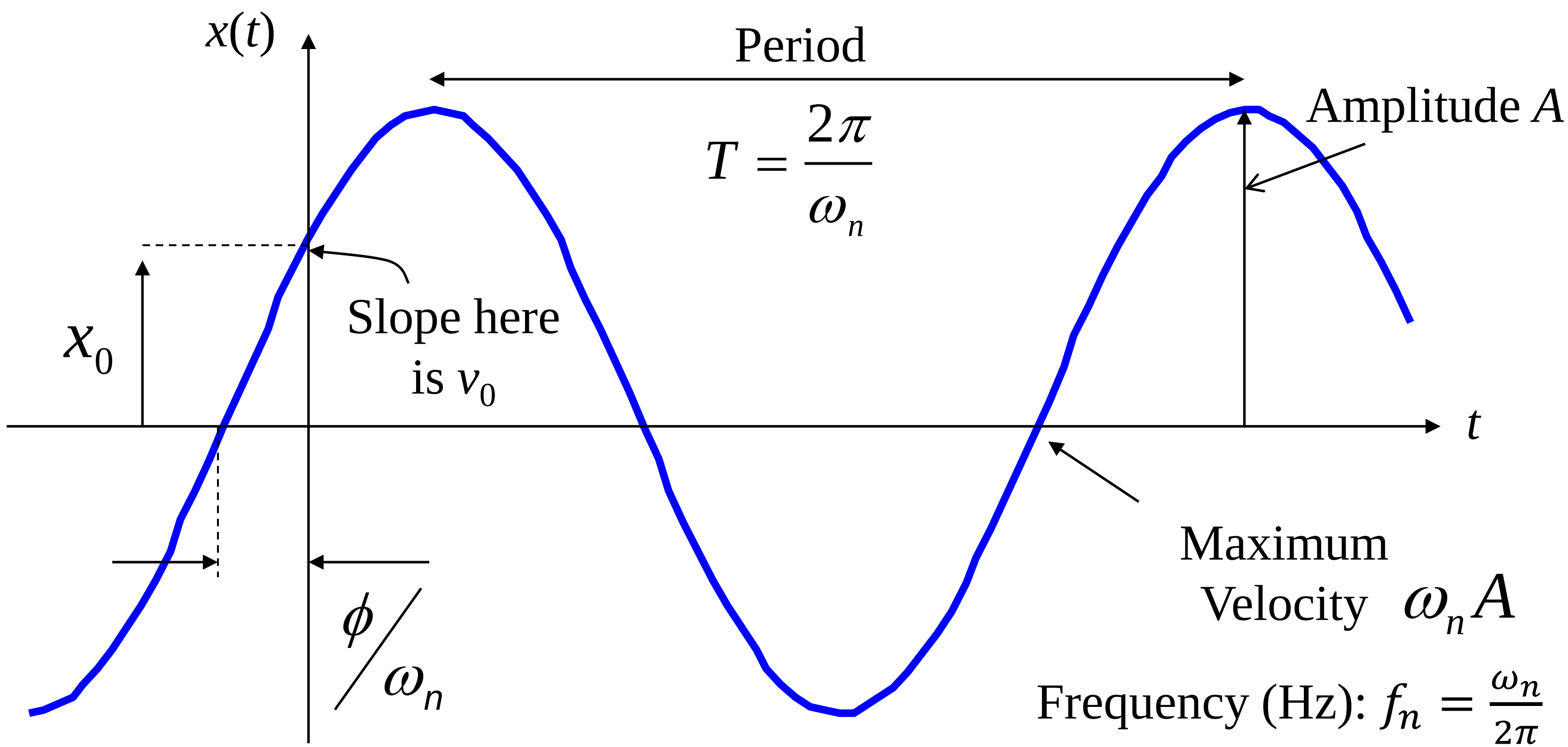
- Acceleration:

$$\ddot{x} = -A\sin\theta = -\omega^2 A\sin\omega\, t = -\omega^2 x$$

Note: frequency $\omega = \frac{\theta}{t}$ (radians/sec) is the constant angular velocity

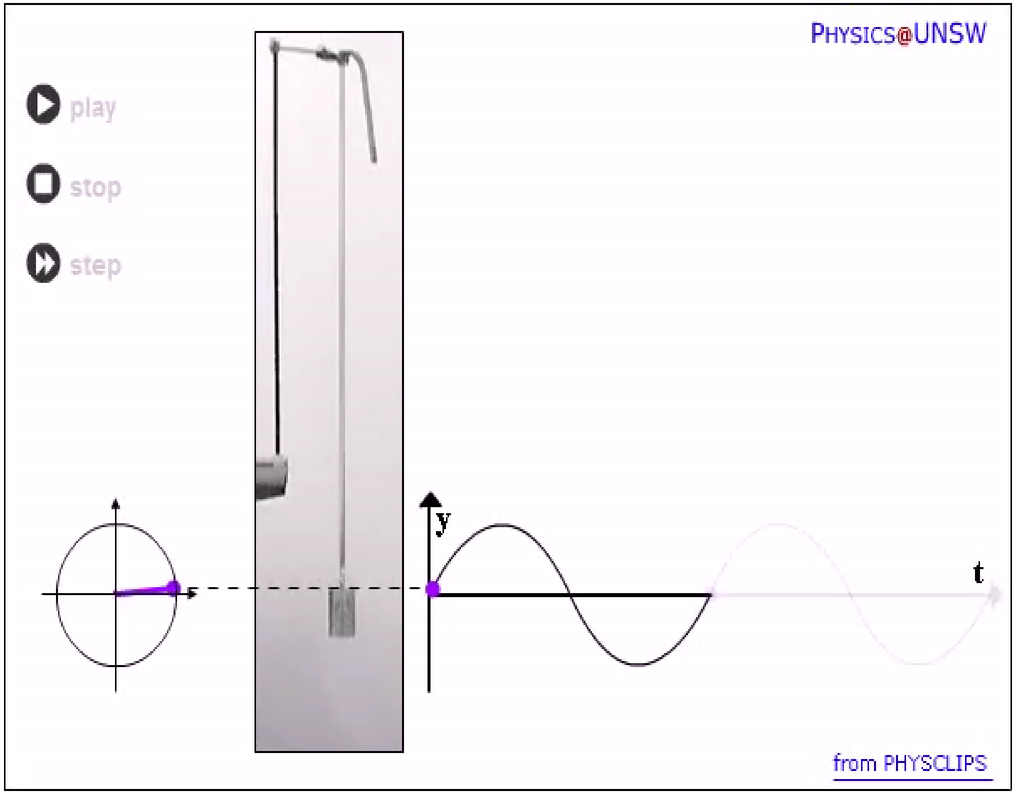


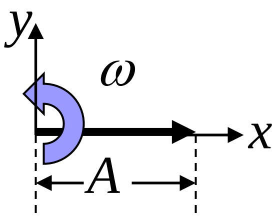
Simple harmonic motion



Displacement: $x = A\sin(\theta + \phi) = A\sin(\omega\, t + \phi)$

Revision - trigonometry

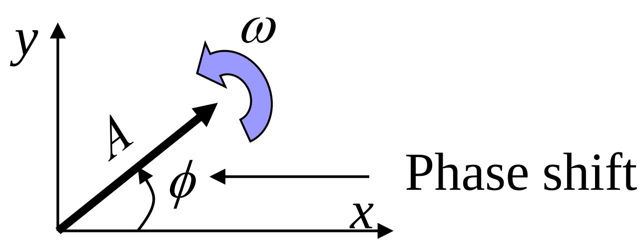




$$y = A\sin(\omega t)$$

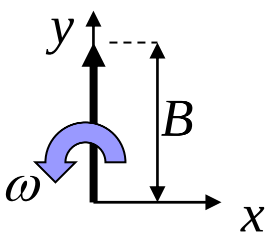
- can be represented as a vector with amplitude "A" rotating counter clockwise with constant angular velocity ω starting from $\theta = 0$

Revision - trigonometry



$$y = A\sin(\omega t + \phi)$$

- can be represented as a vector with amplitude "*A*" rotating counter clockwise with constant angular velocity ω starting from $\theta = \phi$

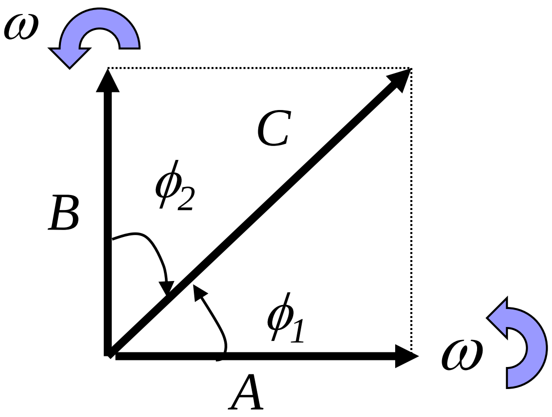


$$y = B\cos(\omega t)$$

- can be represented as a vector with amplitude "*B*" rotating counter clockwise with constant angular velocity ω starting from $\theta = 90°$

Revision - trigonometry

$$x(t) = A\sin(\omega t) + B\cos(\omega t)$$



- This can be viewed as 2 vectors rotating at the same angular velocity ω
- The combination can be viewed as a single rotating vector of magnitude C:

$$x(t) = A\sin(\omega t) + B\cos(\omega t) = C\sin(\omega t + \phi_1) \text{ or}$$

$$x(t) = A\sin(\omega t) + B\cos(\omega t) = C\cos(\omega t - \phi_2)$$

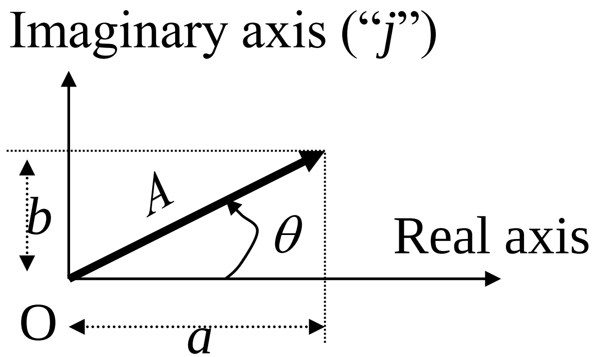
- $C = \sqrt{A^2 + B^2}$
- $\phi_1 = \tan^{-1}(B/A)$
- $\phi_2 = \tan^{-1}(A/B)$

Revision - trigonometry

The rotating vector can also be represented as a complex number:

$$z = a + bj = Ae^{j\theta} = Ae^{j\omega t}$$

- $j = \sqrt{-1}$
- Amplitude: $A = \sqrt{a^2 + b^2}$
- Phase angle: $\phi = \tan^{-1}(b/a)$



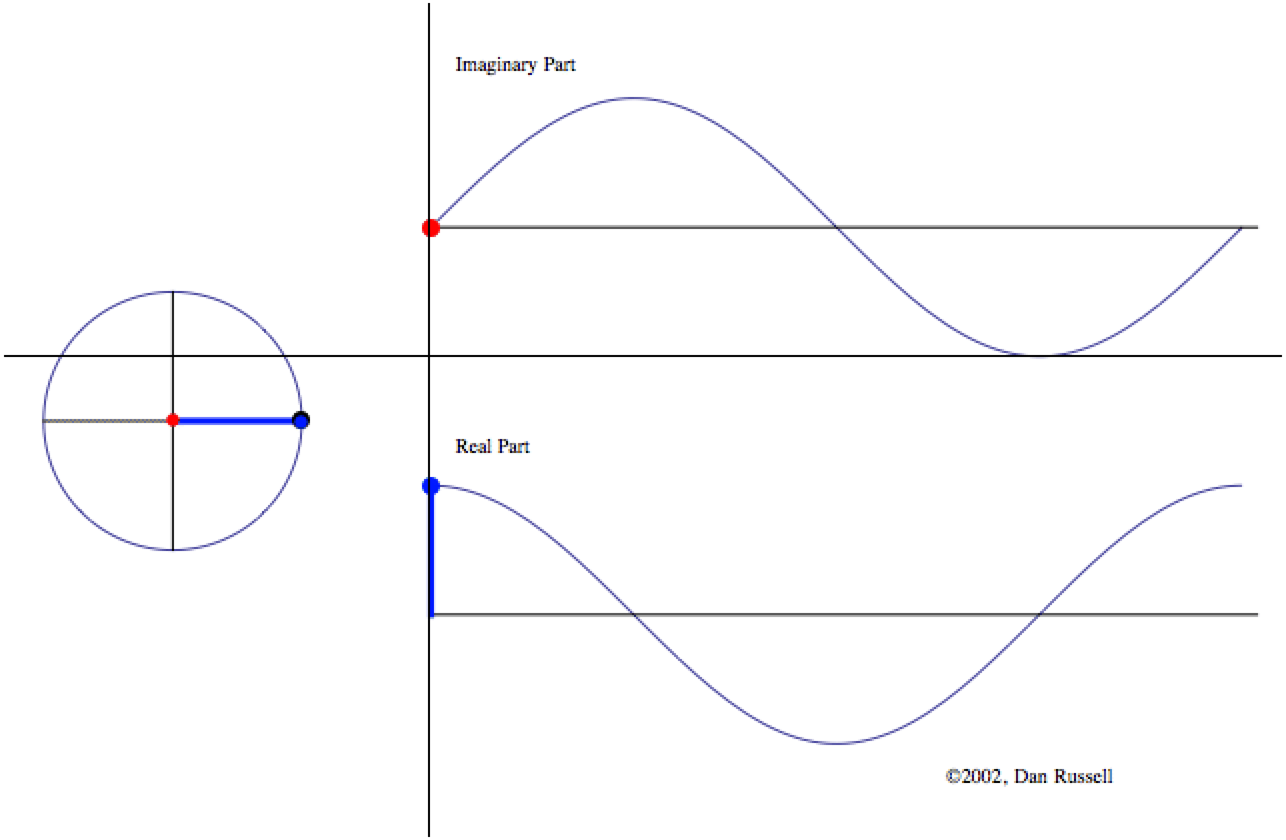
Note:

- $e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$
- $Ae^{j\theta} = Ae^{j\omega t} = A\cos(\omega t) + jA\sin(\omega t)$

Let $z_1 = a_1 + jb_1 = A_1e^{J\theta_1}$ and $z_2 = a_2 + jb_2 = A_2e^{j\theta_2}$

- $z_1 \pm z_2 = A_1e^{j\theta_1} + A_2e^{j\theta_2} = (a_1 \pm a_2) + j(b_1 \pm b_2)$
- $z_1z_2 = A_1A_2e^{j(\theta_1+\theta_2)}$
- $\frac{z_1}{z_2} = \frac{A_1}{A_2}e^{j(\theta_1-\theta_2)}$

Revision - trigonometry



Example 1

Find the sum of two harmonic motions $x_1 = 10\sin(\omega t)$ and $x_2 = 15\sin(\omega t + 2)$

Note that the 2 motions represented as vectors are rotating at the same angular velocity. Represent the harmonic motions as:

- ❖ $10e^{j\omega t} = 10\cos(\omega t) + j10\sin(\omega t) \Longrightarrow x_1 = \mathrm{Im}\left[10e^{j\omega t}\right]$
- ❖ $15e^{j(\omega t+2)} = 15\cos(\omega t + 2) + j15\sin(\omega t + 2) \Longrightarrow x_2 = \mathrm{Im}\left[15e^{j(\omega t+2)}\right]$

At time $t = 0$:

$$\begin{aligned} &10e^{j\omega t} + 15e^{j(\omega t+2)} \\ &= (10\cos(0) + j10\sin(0)) + (15\cos(2) + j15\sin(2)) \\ &= (10 - 6.24222) + j(0 + 13.6395) \\ &= (3.7578) + j(13.6395) = Ae^{j(\omega t+\theta)} \end{aligned}$$

- ❖ $A = \sqrt{(3.7578)^2 + (13.6395)^2} = 14.1477$
- ❖ $\theta = \tan^{-1}\left(\frac{13.6395}{3.7578}\right) = 1.302$ rad
- ❖ $x = x_1 + x_2 = \mathrm{Im}\left[Ae^{j(\omega t+\theta)}\right] = 14.1477\sin(\omega t + 1.302)$

Example 1

Note that all of the following representations are equivalent:

❖ Magnitude and phase form:

$$x = A\sin(\omega t + \phi)$$

❖ Cartesian form:

$$x = A\sin(\omega t) + B\cos(\omega t)$$

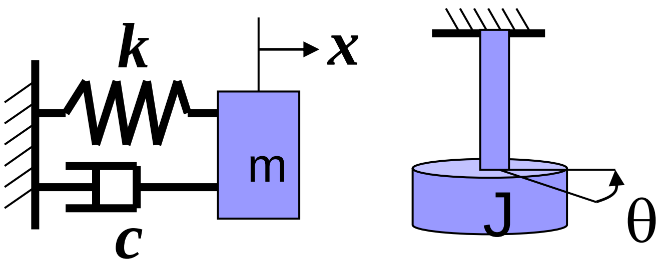
❖ Polar form:

$$x = a_1 e^{j\omega t} + a_2 e^{-j\omega t}$$

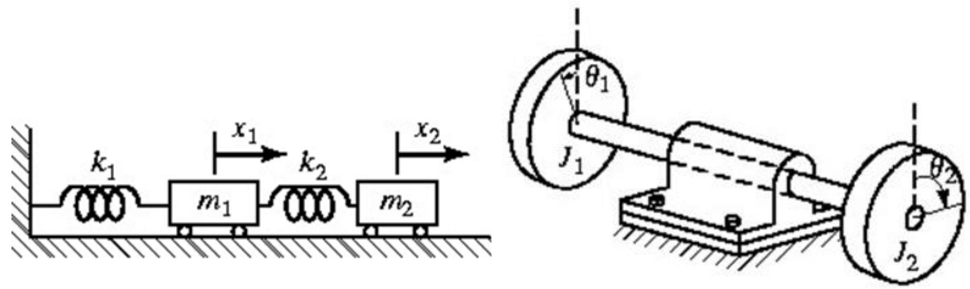
- Each represents the same information
- Each is useful in different situations
- Each gives the same solution

Review – degree of freedom

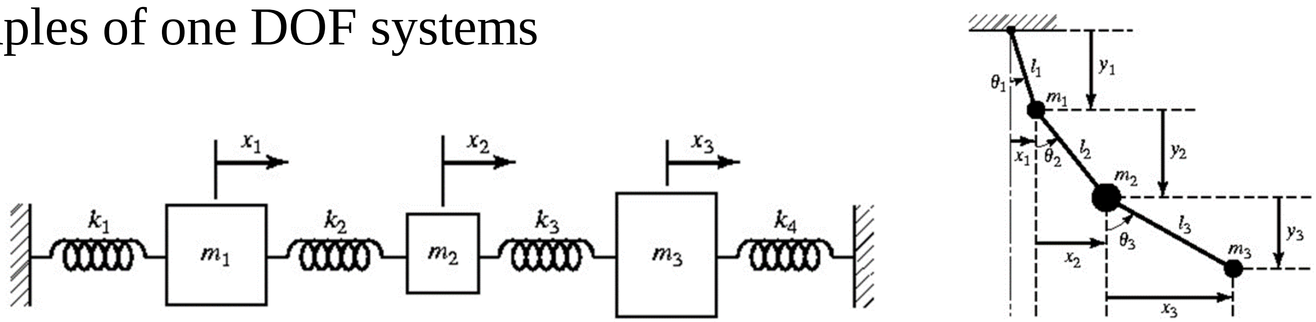
Degree of Freedom (DOF) = minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time



Examples of one DOF systems



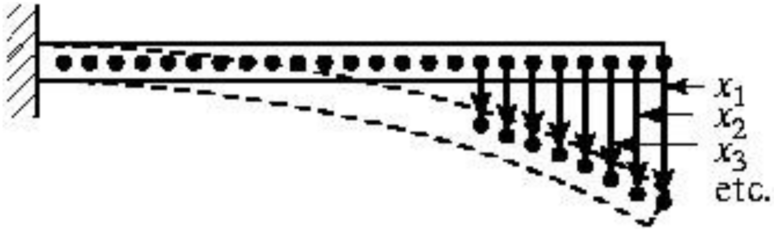
Examples of two DOF systems



Examples of three DOF systems

Review – degree of freedom

- More accurate results obtained by increasing number of degrees of freedom
- Infinite number of degrees of freedom system are termed continuous or distributed systems
- Finite number of degrees of freedom are termed discrete or lumped parameter systems



Example of a continuous system

Free vs forced vibration

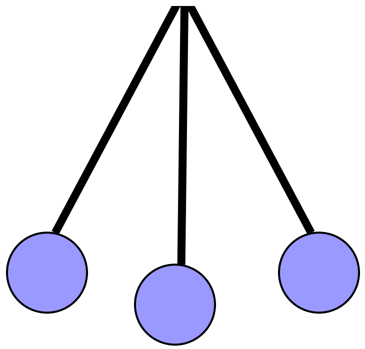
Vibration = any motion that repeats itself after an interval of time

- Free vibration: system is left to vibrate on its own after an initial disturbance with no external force acting on the system. It involves the transfer of potential energy to kinetic energy & vice versa (Example: simple pendulum with initial displacement)
- Generally includes 3 mechanical elements

1. Means to store kinetic energy (inertia elements)
2. Means to store potential energy (spring elements)
3. Means to dissipate energy (damper elements)

- Forced vibration: the system, which can be modelled with inertia, spring and damper elements, is subjected to an oscillating external force (Example: washing machine)

Example: Simple Pendulum



Review – inertia elements

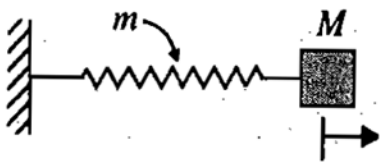
TYPE	LOAD	ENERGY
Translational	$F = m\ddot{x}$	$KE = \frac{1}{2} m\dot{x}^2$
Rotational	$T = I\ddot{\theta}$	$KE = \frac{1}{2} I\dot{\theta}^2$

Inertia is an element associated with kinetic energy

- Force F
- Torque T
- Mass m
- Mass moment of inertia I
- Linear velocity $\dot{x}$ and linear acceleration $\ddot{x}$
- Angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$
- Kinetic energy KE

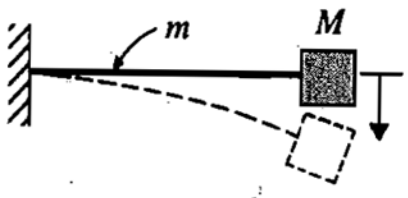
Review – inertia elements

Equivalent mass



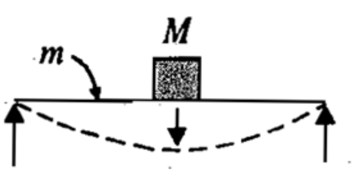
Mass (M) attached at end of spring of mass m

$$m_{eq} = M + \frac{m}{3}$$



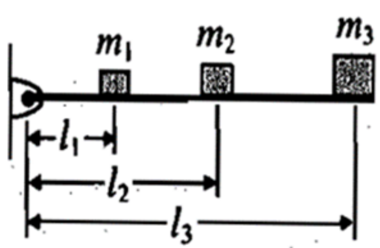
Cantilever beam of mass m carrying an end mass M

$$m_{eq} = M + 0.23\,m$$



Simply supported beam of mass m carrying a mass M at the middle

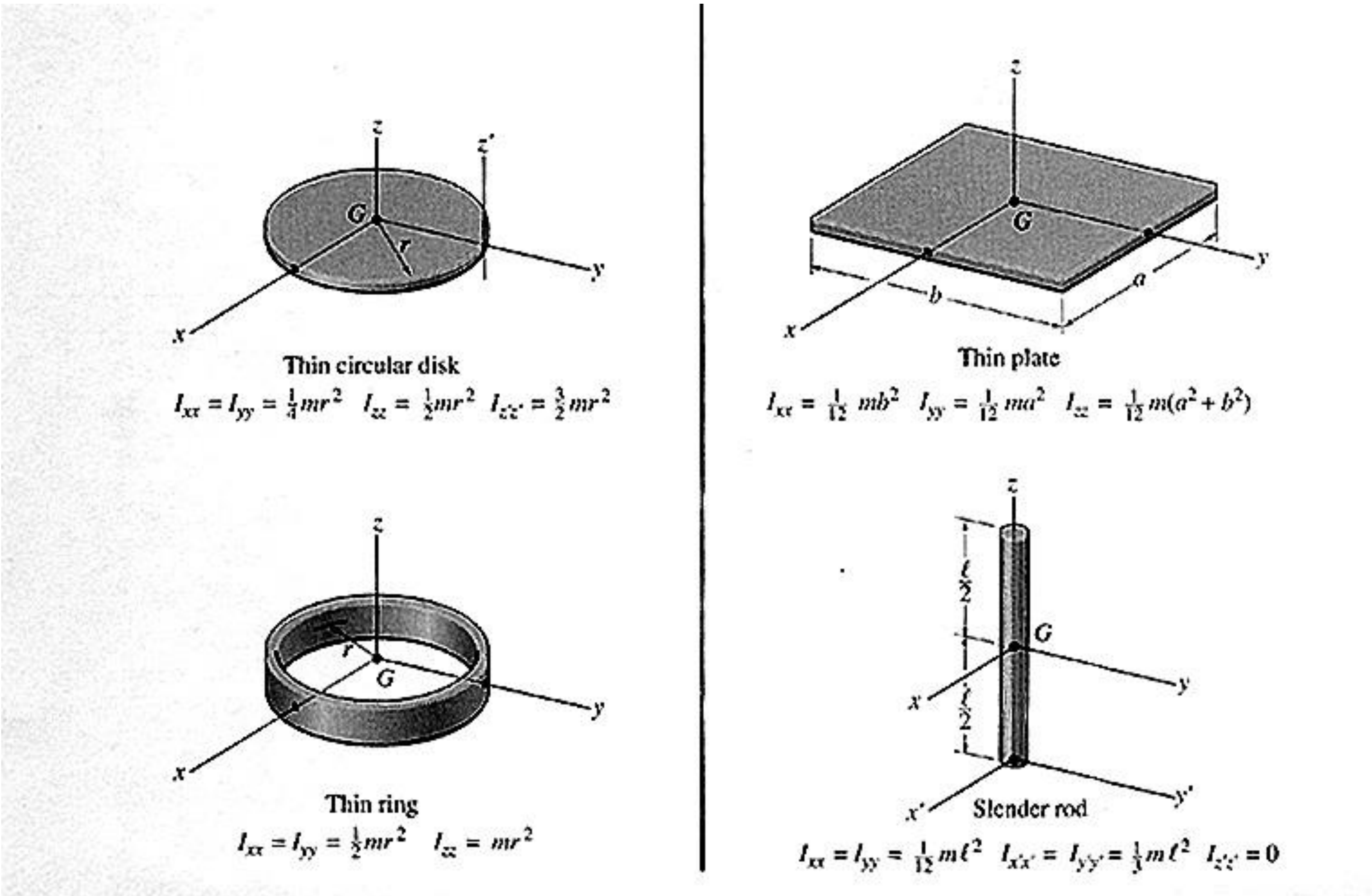
$$m_{eq} = M + 0.5\,m$$



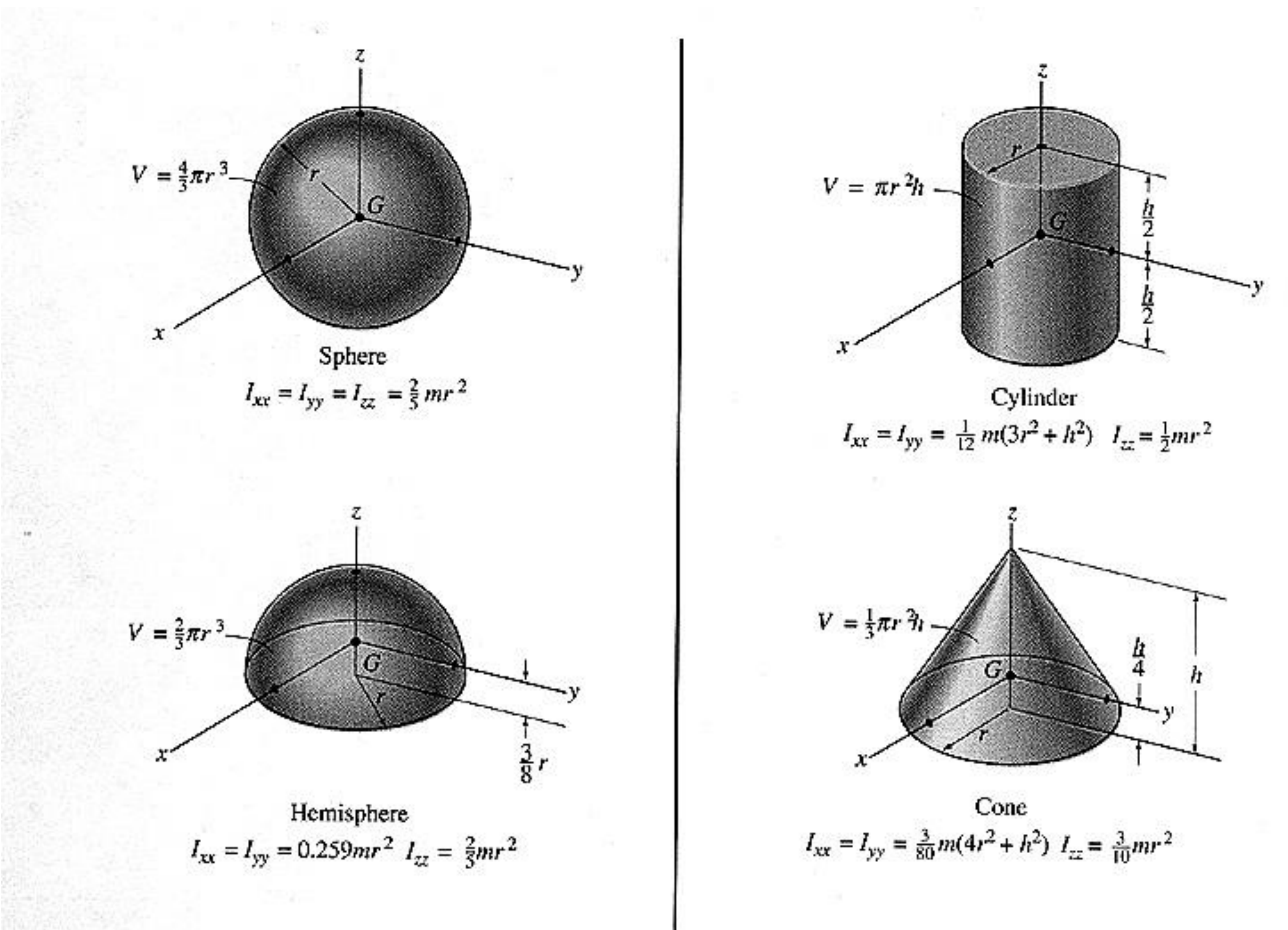
Masses on a hinged bar

$$m_{eq_1} = m_1 + \left(\frac{l_2}{l_1}\right)^2 m_2 + \left(\frac{l_3}{l_1}\right)^2 m_3$$

Review – inertia elements



Review – inertia elements



Review – inertia elements

Parallel axis theorem:
If the mass moment of inertia through the mass center I_G is known, then the mass moment of inertia about any parallel axis I_A through point "*A*" can be found using

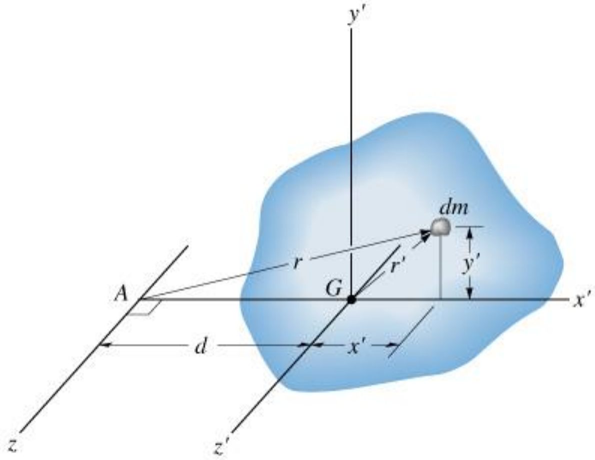
$$I_A = I_G + md^2$$

- I_G = moment of inertia about the axis passing through the mass center
- I_A = moment of inertia about any parallel axis through point "*A*"
- m = total mass of the body
- d = distance between the two parallel axes

Mass moment of inertia can also be expressed as:

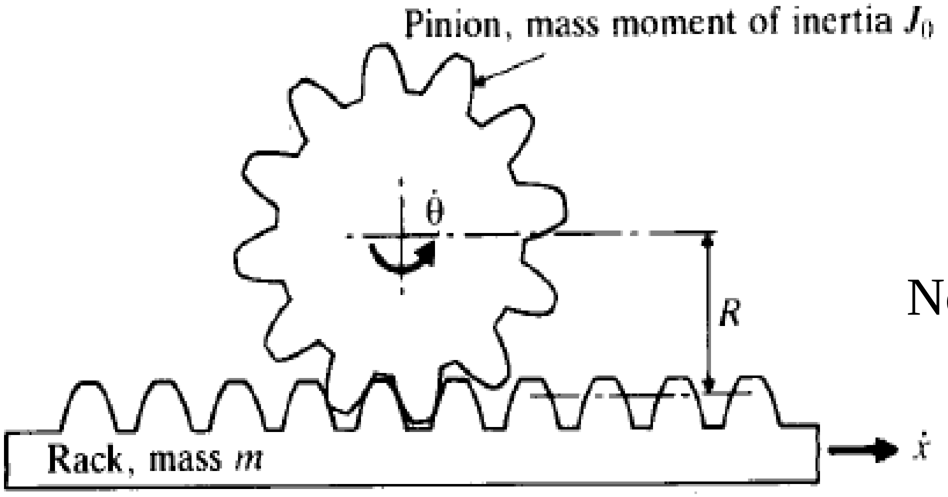
$$I_G = mk^2$$

- Radius of gyration = k,



Example 1

Find the equivalent mass of the coupled translational and rotational rack-pinion system



Note: $\dot{x} = R\dot{\theta}$

The kinetic energy of the system is:

$$KE = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\left(\frac{\dot{x}}{R}\right)^2 = \frac{1}{2}\left(m + \frac{J_0}{R^2}\right)\dot{x}^2 = \frac{1}{2}m_{eq}\dot{x}^2$$

Equivalent mass is $m_{eq} = m + \frac{J_0}{R^2}$